New Method for Eliminating the Statistical Bias in Highly Turbulent Flow Measurements

Shin-ichi Nakao* and Yoshiya Terao†
National Research Laboratory of Metrology, Ibaraki, Japan
and
Kei-ichiro Hirata‡
Kitakyushu Industrial Research Institute, Fukuoka, Japan

A simple method was developed for eliminating statistical bias which can be applied to highly turbulent flows with the sparse and nonuniform seeding conditions. Unlike the method proposed so far, a weighting function was determined based on the idea that the statistical bias could be eliminated if the asymmetric form of the probability density function (pdf) of the velocity data were corrected. Moreover, the data more than three standard deviations away from the mean were discarded to remove the apparent turbulent intensity resulting from noise. The present method was applied to data obtained in the wake of a block, which provided local turbulent intensities up to about 120%. It was found to eliminate the statistical bias with high accuracy.

Nomenclature

U_0 = reference velocity
$\bar{U}$ = local mean velocity
u'/U_0 = turbulent intensity
$u'/\bar{U}$ = local turbulent intensity
G = weighting function
N = number of sampling data
U_i, V_i = ith velocity realization in X and Y directions, respectively

Subscripts

en = ensemble average
1-D = one-dimensional method
2-D = two-dimensional method
new = present method

I. Introduction

THE laser Doppler velocimeter (LDV) is a nonintrusive velocimeter with some remarkable features. However, when it is operated in the individual realization mode, there appears a problem that the probability of a higher velocity particle is more than that of a lower velocity because the particle distributes randomly in space, not in time, and its velocity depends on the instantaneous velocity. Consequently, the ensemble-averaged velocity becomes larger than the real value. This is called the statistical bias, and the higher the turbulent intensity, the larger the statistical bias.

This statistical bias was first studied by McLaughlin and Tiederman,[1] who indicated theoretically that this bias could be eliminated by using the inverse of the instantaneous velocity vector as the weighting function; they proposed the simplified weighting function $1/|U_i|$, where U_i is the ith velocity realization. This method (one-dimensional method) is very simple but is valid only in flows with constant concentration of seeding particles and in near-one-dimensional flowfields (that is, flows with a predominant direction similar to boundary-layer flow). The recent development of the LDV system has enabled researchers to measure two or three velocity components simultaneously and then to obtain the weighting function, proposed theoretically, which can totally eliminate the statistical bias. But the multicomponent LDV system is very expensive, and there is still a demand for the bias-eliminating method applicable to the results of the one-component LDV system.

The inverse of the instantaneous velocity is considered to correspond to the time that a particle takes to pass through the measuring volume, which is called the residence time. Then, Hoesel and Rodi[2] suggested using this time as the weighting function. Another type of time, such as the interarrival time, also is proposed to use as the weighting function.[3] In these approaches, the assumption that the particle concentration is uniform is indispensable for reasonably eliminating the statistical bias. Also, it is not always easy to measure quantities such as the residence time accurately.

The constant time interval sampling method (periodic sampling method), which was developed by Stevenson et al.[4] and analyzed by Erdmann and Tropea[5] and others, is considered to be one of the most reliable methods to eliminate the statistical bias. However, this method also requires certain conditions to obtain reasonable results: specifically, a high concentration of seeding particles and a long sampling time. The satisfaction of these conditions is difficult for measurements in the engineering field and even in the laboratory. Moreover, the relationship among the sampling interval, the turbulent time scale, and the arrival rate has not yet been correctly confirmed.[6]

Good seeding conditions, both high and uniform concentration, are indispensable to all methods discussed so far in order to reasonably eliminate statistical bias. There has been no reliable method that can be applied to flows with poor seeding conditions, because the concentration of particles is sparse and not uniform. The purpose of this paper is to develop a new, simple method to eliminate statistical bias and to produce reasonable results on the application of mean velocity and turbulent intensity to highly turbulent flows with poor seeding conditions. The present method was applied to the results of flow with local turbulent intensity up to about 120%. It was confirmed that the statistical bias could be eliminated with high accuracy.

Received Feb. 21, 1986; revision received Aug. 26, 1986.

*Senior Research Scientist, Fluid Measurement Section. Member AIAA.

†Research Scientist, Fluid Measurement Section.

‡Research Scientist.

II. Principle of the New Method

In turbulent flow measurements, the mean velocity calculated by the usual ensemble average becomes larger than the real value obtained by the statistical bias. This means that the probability density function (pdf) of the individual measurement P_b is biased toward the higher velocity regions and therefore is different from the correct pdf of the velocity P_c. These pdf's are joined by using the weighting function G as follows: $P_c = G * P_b$. Hoesel and Rodi[2] called $1/G$ the relative sampling probability; G is generally considered to be a function of the velocity components, velocity vector, particle concentration, etc. Several forms of the weighting function G have been proposed since the study of McLaughlin and Tiederman,[1] but most of them are based on their study.

In this paper, unlike the methods proposed previously, the determination of the weighting function G is directly related to the form of the pdf. Namely, the pdf of sampling data P_b deviates from the symmetric form as the Gaussian-like form P_c, which should be obtained if the data are sampled randomly in time and there is no statistical bias. Accordingly, if the deviation of P_b from the symmetric form can be corrected, the statistical bias can be eliminated. On the basis of this idea, the following weighting function was finally introduced by trial and error:

$$G_i = \left[1 + (\overline{U}_{en} - U_i)/\overline{U}_{en}\right]^{1.5}$$

where $\overline{U}_{en}$ is the mean velocity calculated by the ensemble average, and U_i is the ith velocity realization. It is easy to find that the inverse of this function behaves similarly to the real relative sampling probability described by Hoesel and Rodi.[2]

Figure 1 shows the typical pdf in turbulent flow measurements. Some smaller probabilities are recognized in the regions of lower and higher velocities, which are enclosed by circles in the figure. These data, which appear strange and seem unreliable, often appear in the measurements of sparsely seeded flows. These data have almost no effect on the determination of the mean velocity, but it is important to remove the unreliable data in order to obtain reasonable turbulent intensity. If the data are similar to the ambiguity noise discussed in detail by George and Lumley,[7] they could be removed by setting a threshold value, since the ambiguity noise is considered to be a type of white noise. That is to say, probabilities smaller than the threshold value would be cut off as the noise. The development of a simple way to remove these unreliable data will be discussed next.

In nonturbulent flow measurements, the turbulent intensity obtained by the LDV shows ~ 2–4%, even where the result of the hot wire is almost zero. This is because the LDV sees the apparent turbulent intensity caused by the ambiguity noise. After several methods were examined to remove the apparent turbulent intensity, the 3σ cutoff method, discarding the data more than 3σ away from the mean, was finally selected because it was very simple and could produce reasonable results, as shown in Fig. 2, although it is not clear why this method can remove the ambiguity noise. The mean velocity was not affected by this operation. As expected, in the case of 2σ cutoff the mean velocity became smaller, and in the case of 4σ cutoff the turbulent intensity was too large.

The present method consists of three steps. First, the sampling data are automatically cut off more than 3σ away from the mean; second, the rest of the data are multiplied by the weighting function; and third, the mean velocity and turbulent intensity are calculated by the following equations:

$$\overline{U}_{en} = \sum_i^N U_i/N$$

$$\sigma = u'_{en} = \left[\sum_i^N (\overline{U}_{en} - U_i)^2 \Big/ N\right]^{\frac{1}{2}}$$

If $(\overline{U}_{en} - 3 \times \sigma) < U_i < (\overline{U}_{en} + 3 \times \sigma)$, then

$$\overline{U}_{new} = \sum_i^n U_i \times G_i \Big/ \sum_i^n G_i$$

$$u'_{new} = \left[\sum_i^n (\overline{U}_{new} - U_i)^2 \times G_i \Big/ \sum_i^n G_i\right]^{\frac{1}{2}}$$

where n is the number of the rest of the data.

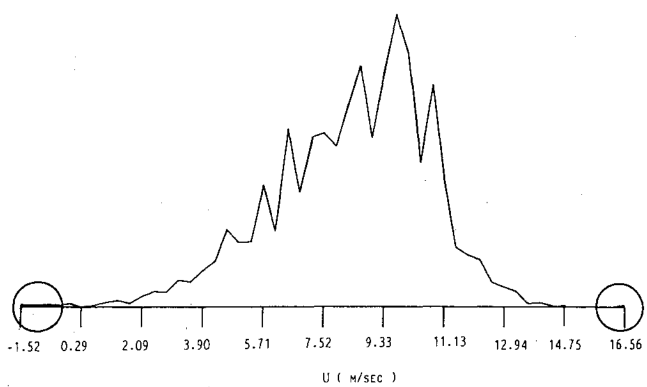

Fig. 1 Typical probability density function of the turbulent flow ($x/H = 3.0$, $y/H = 1.75$, and $z/H = 0.4$).

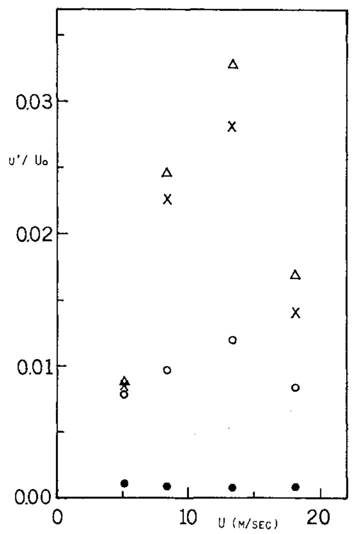

Fig. 2 Turbulent intensity in nonturbulent flows (△: ensemble average, ×: one-dimensional method, ○: present method, ●: hot wire).

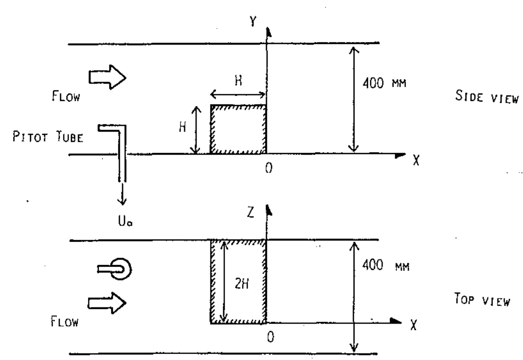

Fig. 3 Experimental geometry of the flowfield with the block ($H = 120$ mm).

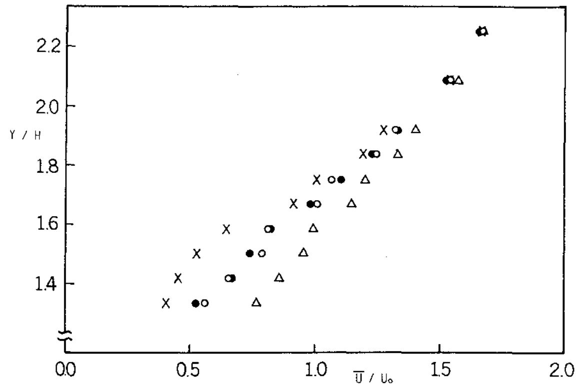

Fig. 4 Distribution of the mean velocity in the plane: $x/H = 3.0$ and $z/H = 0.5$ (○: $\overline{U}_{new}/U_0$, △: $\overline{U}_{en}/U_0$, ×: $\overline{U}_{1\text{-D}}/U_0$, ●: $\overline{U}_{2\text{-D}}/U_0$).

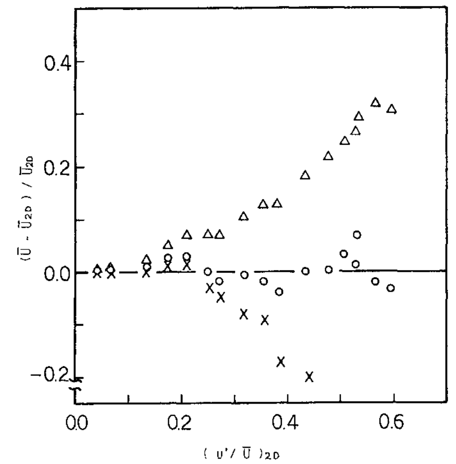

Fig. 5 Comparison of biased velocities with velocities obtained by using the present method; range of below 60% local turbulent intensity (○: present method, △: ensemble average, ×: one-dimensional method).

The largest problem in the assessment of the new method for eliminating the statistical bias is that there are no reliable velocimeters that can be used as a standard for comparison in highly turbulent flow measurements. Johnson et al.[8] reported that the two-dimensional correction method using the two-component LDV system (two-dimensional method), which is the extension of the one-dimensional method, could produce good results on both the mean velocity and the turbulent intensity. Then they also used the two-dimensional method as the standard for comparison with the present results. But here it must be noted that the two-dimensional method becomes unreliable in the measurement of flows with almost zero velocity components, such as separating flows.

III. Experimental Apparatus

The measurements to assess the new method were made in the wake of a three-dimensional block, like a building, in which different and higher turbulent levels can easily be provided. The schematic layout is shown in Fig. 3. The reference velocity U_0 was checked at each measurement by the pitot tube set at the entrance of the measuring section. U_0 was about 6.7 m/s and its fluctuation was under $\pm 0.5\%$. The Reynolds number based on the block height ($H = 12$ cm) was about 5.4×10^4.

The LDV used in this paper is the three-beam, two-component system (DANTEK 55X series) with the frequency shifter (DANTEK 55N10) using the counter processors (DANTEK 55L90a). Light source is the 3-W argon ion laser (Spectra-Physics model 2020-03). The diameter and length of the measuring volume are approximately 0.15 and 3.5 mm, respectively. All experiments were carried out in the forward-scattering mode with a 1-3 MHz shift. After the sampling data are stored in the buffer memory, they are fed into the 16-bit microcomputer NEC PC 9801-E for data processing and analysis. The pdf of the velocity was always monitored on the display and, if necessary, was drawn by the plotter (HP 7470A). The number of sampling data was $5K$ ($K = 1024$) at each measurement. The data rate was changeable but was ~ 20 Hz on the average through all measurements in the one-component mode.

No special seeding generator was used. Some incense sticks were burned at the entrance of the wind tunnel, which could produce particles of about 1 μm diam and was enough to obtain the data rate mentioned previously. The data rate can be increased easily by burning more sticks, but the concentration of seeding particles is not uniform because the burning of sticks is not controlled.

When the LDV is used in the two-component mode, the data sampling is controlled as follows. If one counter processor receives the effective signal from the particle within the time interval Δt after the other processor, it is assumed that these signals occurred from the same particle. Therefore, they are handled as a pair in the following analysis. The time interval Δt can be changed to four settings (400, 200, 100, and 50 μs), but in this paper Δt was usually set at 200 μs because there were no remarkable differences in the results between $\Delta t = 50$ and 200 μs. The data rate in the two-component mode was over 200 Hz but was not uniform.

IV. Results and Discussion

The results of applying the present method to the measurements in the wake of the block were compared with those of the one-dimensional method and the two-dimensional method. They are generally written with the weighting function G_i as follows:

$$\overline{U} = \sum_i^N U_i \times G_i \Big/ \sum_i^N G_i$$

$$u' = \left[\sum_i^N (\overline{U} - U_i)^2 \times G_i \Big/ \sum_i^N G_i \right]^{\frac{1}{2}}$$

One-dimensional method:

$$G_i = 1/|U_i|$$

Two-dimensional method:

$$G_i = 1/\left(U_i^2 + V_i^2\right)^{\frac{1}{2}}$$

where $G_i = 1$ in the usual ensemble average.

Figure 4 shows the typical distribution of the mean velocity $\overline{U}$. It is found that the results of the present method are in very good agreement with those of the two-dimensional method. On the other hand, the one-dimensional method overcorrects, and the results of the ensemble average become insignificant in the region under about $Y/H = 1.7$, where the local turbulent intensity is over 60%. Figures 5 and 6 indicate the relationship between the magnitude of the statistical bias and the local turbulent intensity: in Fig. 5 below 60% local turbulent intensity, and in Fig. 6 over 40% local turbulent intensity. In Fig. 5, the magnitude of the bias is almost equal to the square of the local turbulent intensity, as the theoretical study estimated. It is found that the present method can eliminate the statistical bias with high accuracy, and the one-dimensional method does not work well at over 30% local turbulent intensity.

The parabolic relation between the magnitude of the bias and the local turbulent intensity is lost in Fig. 6; namely, the increasing rate of the statistical bias decreases at over 60%

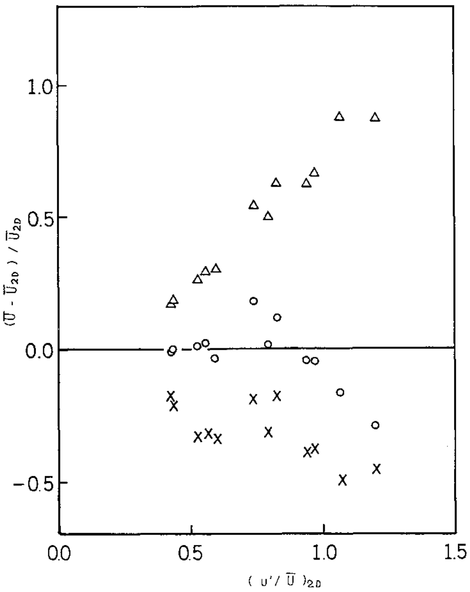

Fig. 6 Comparison of biased velocities with velocities obtained by using the present method; range of over 40% local turbulent intensity (symbols same as Fig. 5).

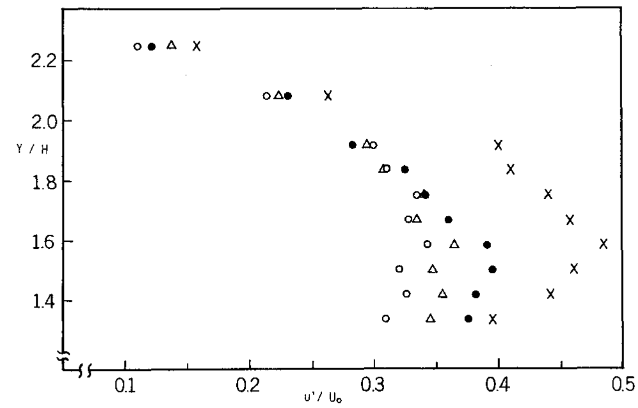

Fig. 7 Distribution of turbulent intensity in the plane: $x/H = 3.0$ and $z/H = 0.5$ (○: u'_{new}/U_0, △: u'_{en}/U_0, ×: $u'_{1\text{-D}}/U_0$, ●: $u'_{2\text{-D}}/U_0$).

local turbulent intensity. This deviation from the parabolic relation is recognized in the theoretical study of Lehmann,[9] which is the extension of the study of McLaughlin et al. Figure 6 shows that the one-dimensional method can no longer provide reasonable results, and the present method works effectively even in the region of over 50% local turbulent intensity. However, the present results are rather scattered in comparison with those in Fig. 5 and show a deviation of about 30% at 120% local turbulent intensity. The experiments were carried out in the recirculating flow region, where accurate measurements are very difficult, and Lehmann's theoretical results also show a deviation of about 20% at the range between 50% and 100% of local turbulent intensity. Accordingly, if these facts are taken into consideration, the results of the present method are considered to have sufficient accuracy within the experimental uncertainty.

Figure 7 shows the distribution of the turbulent intensity in the case of Fig. 5. The one-dimensional method is unreliable for all regions. The new method, the two-dimensional method, and the ensemble average show good agreement among each other until about $Y/H = 1.7$. Their results are scattered in the region under $Y/H = 1.7$, and the difference between the present method and the two-dimensional method is about 25% at maximum. Figure 8 shows the deviation of the local turbulent intensity from the results of the two-dimensional method at over 40% local turbulent intensity. It is found that the results of the one-dimensional method are quite large, and the ensemble-averaged ones are small and show a deviation of about −30% on the average, which is very similar to the results of Lehmann's study. The deviation of the present results is fairly small in comparison with those of the other methods, but the present results show scatter of ±30% at over 60% local turbulent intensity. Such scatter is not recognized in the results of the ensemble average, although it was calculated from the same sampling data. It is estimated that this scatter comes from the characteristics of the weighting function, which is closely related to the pdf form of sampling data. Accordingly, the scatter of the results is considered to be inevitable in the application of the present method to highly turbulent flow, the local turbulent intensity being over 60%.

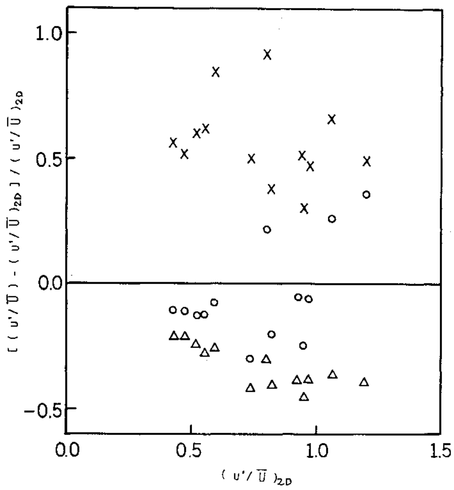

Fig. 8 Deviation of local turbulent intensity from the standard (symbols same as Fig. 5).

Finally, there is a question that the two-dimensional method may not give reasonable results in the region under $Y/H = 1.7$ in Fig. 7 for the reason discussed in Sec. II. The forms of the pdf obtained in that region by the two-dimensional method were reasonable and did not show any abnormalities. Therefore, it is considered that the two-dimensional method can bring about reasonable results of turbulent intensity and mean velocity, even in such a region.

V. Conclusions

A simple method was developed to eliminate statistical bias. This method can be applied to highly turbulent flows with poor seeding conditions, which are sparse and not uniform. The method was used to determine the results at the wake of the three-dimensional block as follows:

1) For mean velocity, the present results showed good agreement with those of the two-dimensional method at the range of below 60% local turbulent intensity.

2) For over 60% and up to 120% local turbulent intensity, the difference between the present results and the two-dimensional method was about 30% at the maximum, but this error is considered to be small enough if experimental uncertainty is taken into consideration.

3) For turbulent intensity, agreement between the present results and the two-dimensional method was very good at the range of below 40% local turbulent intensity.

4) In flows with over 60% local turbulent intensity, the present method indicates a scatter of about ±30% in the results of the turbulent intensity.

References

[1]McLaughlin, D.K. and Tiederman, W.G., "Biasing Correction for Individual Realization of Laser Anemometer Measurements in Turbulent Flows," *The Physics of Fluids*, Vol. 16, Dec. 1973, pp. 2082–2088.

[2]Hoesel, W. and Rodi, W., "New Biasing Elimination Method for Laser Doppler Velocimeter Counter Processing," *Review of Scientific Instruments*, Vol. 48, July 1977, pp. 910–919.

[3]Durao, D.F.G., Laker, J., and Whitelaw, J.H., "Bias Effects in Laser Doppler Anemometry," *Journal of Physics E: Scientific Instruments*, Vol. 13, April 1980, pp. 442–445.

[4]Stevenson, W.H., Thompson, H.D., and Craig, R.R., "Laser Velocimeter Measurements in Highly Turbulent Recirculating Flows," *Transactions of the ASME, Journal of Fluid Engineering*, Vol. 106, June 1984, pp. 173–180.

[5]Erdmann, J.C. and Tropea, C., "Turbulence-Induced Statistical Bias in Laser Anemometry," *Proceedings of the 7th Symposium on Turbulence*, Univ. of Missouri–Rolla, 1981, pp. 129–138.

[6]Adams, E.W., Easton, J.K., and Johnston, J.P., "An Examination of Velocity Bias in a Highly Turbulent Separated and Reattaching Flow," *2nd International Symposium on Application of LDA to Fluid Mechanics*, Lisbon, 1984.

[7]George, W.K. and Lumley, J.L., "The Laser Doppler Velocimeter and Its Application to the Measurements of Turbulence," *Journal of Fluid Mechanics*, Vol. 60, Pt. 2, 1973, pp. 321–362.

[8]Johnson, D.A., Modrarress, D., and Owen, F.K., "An Experimental Verification of Laser Velocimeter Sampling Bias and Its Correction," *Transactions of the ASME, Journal of Fluid Engineering*, Vol. 106, March 1984, pp. 5–12.

[9]Lehmann, B., "A Spatially Working Model to Correct the Statistical Biasing Error of More Directional One Component LDA Measurements," *International Symposium on Application of LDA to Fluid Mechanics*, Lisbon, 1982.